

## Surface Area of Pyramids, Cylinders, Spheres, &amp; Cones

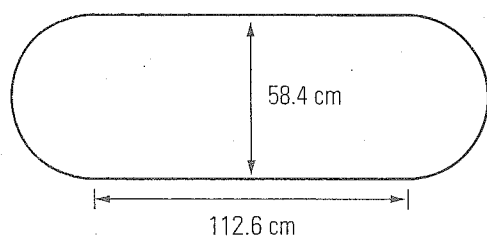
## 3.2

## REVIEW: WORKING WITH CIRCLES

In this section, you will need to calculate the circumference and the area of circles.

## Example 1

Find the area of the following figure.



## SOLUTION

This can be divided into parts: a rectangle 112.6 cm long and 58.4 cm wide, and two semicircles, each with a diameter of 58.4 cm. The two semicircles will make a full circle.

Calculate the area of the rectangle and the circle.

$$A_r = \ell w$$

$$A_r = 112.6 \times 58.4$$

$$A_r = 657.84 \text{ cm}^2$$

$$A_c = \pi r^2$$

$$A_c = \pi(58.4 \div 2)^2$$

$$A_c = \pi(29.2)^2$$

$$A_c \approx 2678.65 \text{ cm}^2$$

$$A = A_r + A_c$$

$$A = 657.84 + 2678.65$$

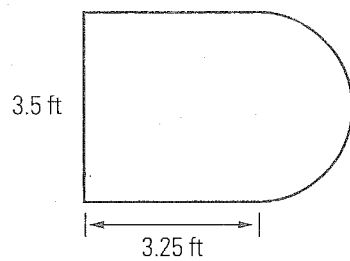
$$A \approx 3336.5 \text{ cm}^2$$

The area is 3336.5 cm<sup>2</sup>.

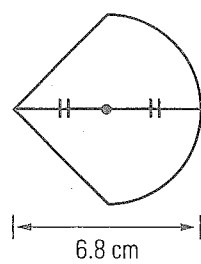
Use the  $\pi$  button on your calculator to calculate the area and circumference. Round your answer to same degree of accuracy (number of decimal places) as are given in the question.

**BUILD YOUR SKILLS**

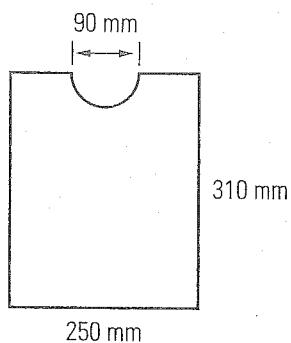
1. Calculate the area of the following figure.



2. Calculate the area of the following figure.

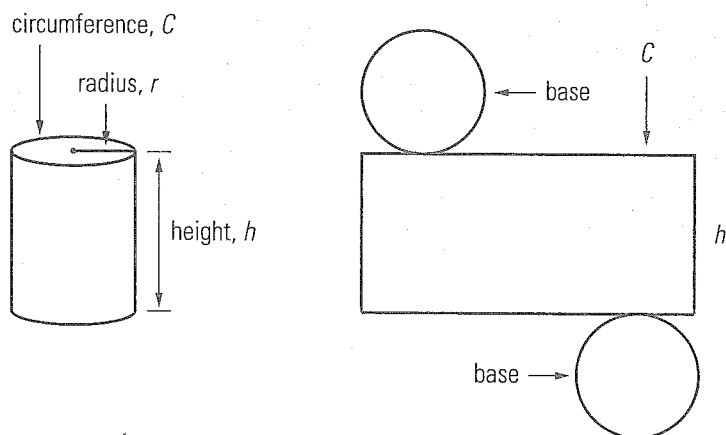


3. Calculate the area of the following figure.



### NEW SKILLS: WORKING WITH THE SURFACE AREA OF CYLINDER

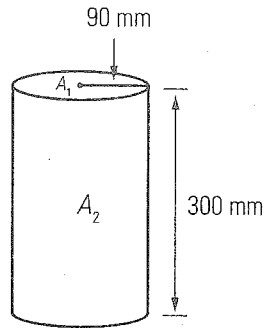
A cylinder is like a prism but it has circular bases. To find the surface area, you have to find the area of the two circles and the area between them. If you draw a net of a cylinder, you will find that it is made up of a rectangle and two circles. The length of the rectangle will be the circumference of the circle, and the width will be the height of the cylinder.



For more details, see page 127 of *MathWorks 11*.

**Example 2**

Find the surface area of a cylinder that has a radius of 90 mm and a height of 300 mm.

**SOLUTION**

First find the area of the circular base of the cylinder.

$$A_1 = \pi r^2$$

$$A_1 = \pi(90)^2$$

$$A_1 \approx 25\,446.9 \text{ mm}^2$$

Next, find the circumference of the cylinder.

$$C = 2\pi r$$

$$C = 2\pi(90)$$

$$C \approx 565.5 \text{ mm}$$

Find the area of the lateral face.

$$A_2 = C \times h$$

$$A_2 = 565.5 \times 300$$

$$A_2 \approx 169\,646.0 \text{ mm}^2$$

Calculate the total surface area.

$$SA = 2(A_1) + A_2$$

$$SA = 2(25\,446.9) + 169\,646.0$$

$$SA = 220\,539.8 \text{ mm}^2$$

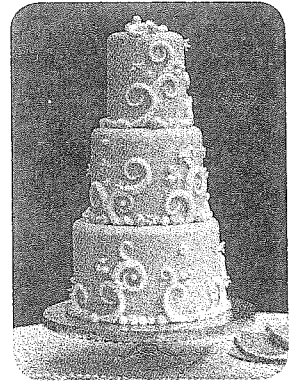
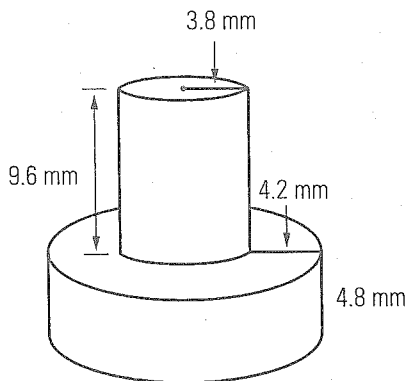
The total surface area is  $220\,539.8 \text{ mm}^2$ .

**BUILD YOUR SKILLS**

4. Find the surface area of a cylindrical tank that has a radius of 1.5 m and a height of 5 m.

5. Find the surface area of a pipe that has a diameter of 4.5 cm and is 18.8 cm long.

6. Find the surface area of the figure below. The upper cylinder is centred on the lower one.



How would you calculate how much icing was needed to cover this cake?

### NEW SKILLS: WORKING WITH THE SURFACE AREA OF PYRAMIDS

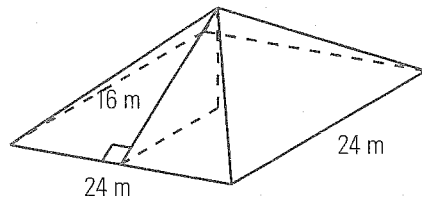
A pyramid is a three-dimensional object with a polygonal base and lateral sides that are triangles. The triangles meet at a point, called the apex. In a right pyramid, the apex is aligned above the centre of the base.

The net of a pyramid will consist of the base plus as many triangles as there are sides to the base.

For more details, see page 127 of *MathWorks 11*.

#### Example 3

Find the surface area of the square-based pyramid below.



#### SOLUTION

The pyramid has a square base that is 24 m by 24 m. The four sides are congruent triangles with a base of 24 m and a height of 16 m.

Calculate the area of the base.

$$A_1 = s^2$$

$$A_1 = 24^2$$

$$A_1 = 576 \text{ m}^2$$

Calculate the area of one of the side triangles.

$$A_2 = \frac{1}{2}(bh)$$

$$A_2 = \frac{1}{2} \times 24 \times 16$$

$$A_2 = 192 \text{ m}^2$$

Calculate the total surface area.

$$SA = A_1 + 4(A_2)$$

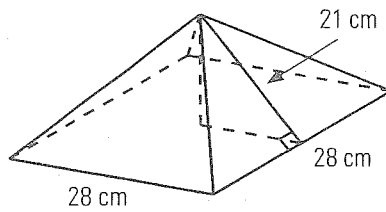
$$SA = 576 + 4(192)$$

$$SA = 1344 \text{ m}^2$$

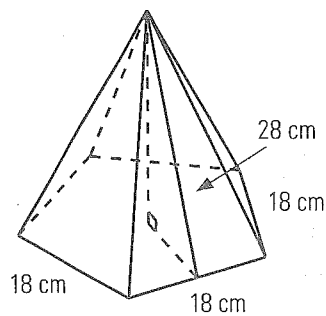
The surface area of the pyramid is  $1344 \text{ m}^2$ .

### BUILD YOUR SKILLS

7. Find the surface area of the square-based pyramid below.



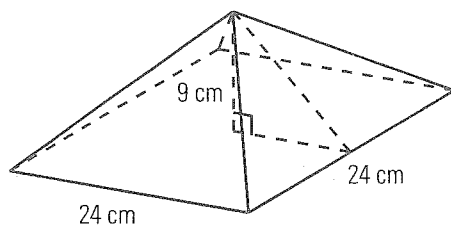
8. A pentagonal-based pyramid is sitting on a table. If the sides of the pentagon are 18 cm and the slant height of the triangles is 28 cm, what is the exposed area of the pyramid?



9. An octagonal-based pyramid has a slant height of 18.7 cm and each side of the octagon measures 12.6 cm. What is the lateral surface area of the pyramid?

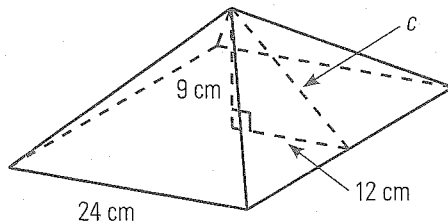
#### Example 4

Find the surface area of the square-based pyramid.



#### SOLUTION

To find the height of the triangles that form the lateral faces—or the slant height of the pyramid—you must use a triangle as indicated below.



You must first find the slant height of the pyramid using the Pythagorean theorem.



The two sides of the triangle will be 9 cm and 12 cm (half the length of a side of the square), and the hypotenuse of this triangle will be slant height of the pyramid.

$$a^2 + b^2 = c^2$$

$$9^2 + 12^2 = c^2$$

$$81 + 144 = c^2$$

$$225 = c^2$$

$$\sqrt{225} = c$$

$$15 = c$$

The surface area of the pyramid is the area of the square base plus the area of the four triangles.

$$A_{\text{square}} = s^2$$

$$A_{\text{square}} = 24^2$$

$$A_{\text{square}} = 576 \text{ cm}^2$$

$$A_{\text{triangle}} = \frac{1}{2}bh$$

$$A_{\text{triangle}} = \frac{1}{2} \times 24 \times 15$$

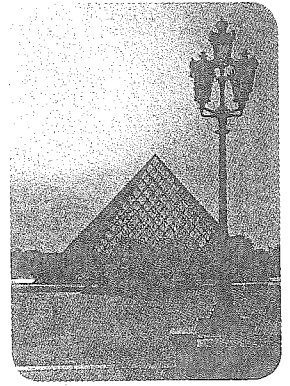
$$A_{\text{triangle}} = 180 \text{ cm}^2$$

$$SA = A_{\text{square}} + 4(A_{\text{triangle}})$$

$$SA = 576 + 4(180)$$

$$SA = 1296 \text{ cm}^2$$

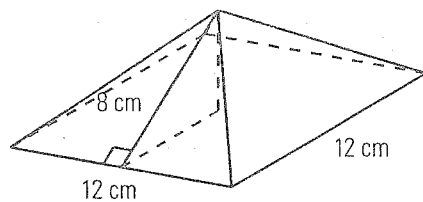
The surface area of the pyramid is 1296 cm<sup>2</sup>.



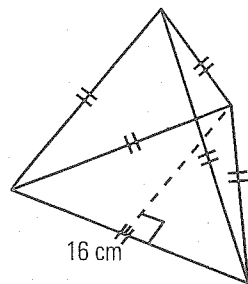
The courtyard of the Louvre Museum in Paris, France, features a large glass pyramid designed by architect I.M. Pei. The Louvre is home to many famous works of art, including the *Mona Lisa* and the *Venus de Milo*.

**BUILD YOUR SKILLS**

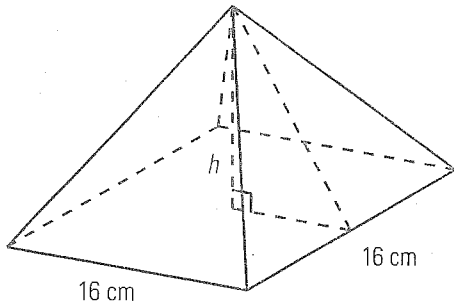
10. Find the total surface area of a square pyramid with a base of 12 cm by 12 cm and a height of 8 cm.



11. A triangular pyramid has faces that are all equilateral triangles. Each side length is 16 cm. What is the surface area of the pyramid?



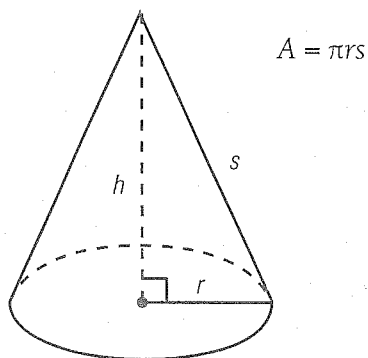
12. If the surface area of the sides of a square-based pyramid is  $680 \text{ cm}^2$  and the side lengths of the square are  $16 \text{ cm}$ , what is the height of the pyramid?



### NEW SKILLS: WORKING WITH THE SURFACE AREA OF CONES

A cone is like a pyramid, but it has a circular base. The net of a cone is a sector of a large circle, and the circular base of the cone.

The surface area of the lateral area of the cone (the area not including the base) can be calculated using this formula, where  $r$  is the radius of the circular base and  $s$  is the slant height of the lateral face:



**Example 5**

Find the surface area of a cone that has a radius of 12 feet and a slant height of 15 feet.

**SOLUTION**

Calculate the area of the circular base.

$$A_1 = \pi r^2$$

$$A_1 = \pi(12)^2$$

$$A_1 \approx 452.39 \text{ sq. ft.}$$

Calculate the area of the lateral surface.

$$A_2 = \pi rs$$

$$A_2 = \pi(12)(15)$$

$$A_2 \approx 565.49 \text{ sq. ft.}$$

Calculate the total surface area.

$$SA = A_1 + A_2$$

$$SA = 452.39 + 565.49$$

$$SA \approx 1017.9 \text{ sq. ft.}$$

The surface area of the cone is about 1017.9 sq. ft.

**ALTERNATIVE SOLUTION**

If you are calculating the total surface area of a cone, you can combine the equations for the base and the lateral surface.

$$SA = \pi rs + \pi r^2$$

$$SA = \pi r(s + r)$$

Use this formula to calculate the surface area.

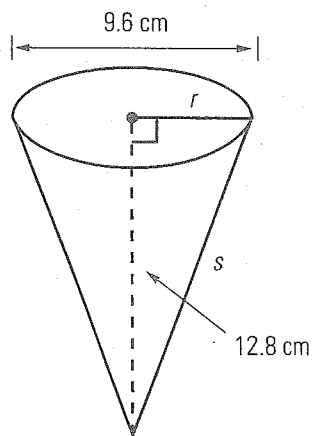
**BUILD YOUR SKILLS**

13. Find the surface area of a cone that has a slant height of 82 cm and a radius of 28 cm.

14. Find the surface area of a cone with a diameter of 13.6 cm and a slant height of 9.8 cm.

**Example 6**

Find the surface area of the cone.



**SOLUTION**

The slant height is not given, so you will first need to calculate it using the Pythagorean theorem. Since the diameter of the cone is 9.6 cm, the radius is 4.8 cm.

$$a^2 + b^2 = c^2$$

$$h^2 + r^2 = s^2$$

$$(12.8)^2 + (4.8)^2 = s^2$$

$$163.84 + 23.04 = s^2$$

$$186.88 = s^2$$

$$\sqrt{186.88} = s$$

$$13.67 \approx s$$

The slant height is approximately 13.67 cm.

Since you are calculating the total surface area, you can use the combined formula used in Example 6.

$$SA = \pi r(s + r)$$

$$SA = \pi(4.8)(13.67 + 4.8)$$

$$SA \approx 278.5 \text{ cm}^2$$

The surface area is approximately 278.5 cm<sup>2</sup>.

**BUILD YOUR SKILLS**

15. Find the total surface area of a cone with a radius of 16 inches and a height of 20 inches.

16. Find the surface area of a cone with a radius of 45.7 mm and a height of 39.7 mm.

17. Find the lateral surface area of a cone whose height is 32 cm and whose diameter is 28 cm.

### NEW SKILLS: WORKING WITH THE SURFACE AREA OF SPHERES

A sphere is like a ball. All points on the sphere are equidistant (of equal distance) from the centre. It is not possible to draw a net of a sphere. The formula for a sphere's surface area depends only on the radius and  $\pi$ . The formula for the surface area of a sphere is:

$$SA = 4\pi r^2$$

For more details, see page 127 of *MathWorks 11*.

#### Example 7

A ball has a surface area of approximately  $9900 \text{ cm}^2$ . What is its radius?

#### SOLUTION

You are given the surface area and need to find the radius. Use the formula for surface area.

$$SA = 4\pi r^2$$

$$9900 = 4\pi r^2$$

$$\frac{9900}{4\pi} = \frac{4\pi r^2}{4\pi}$$

$$\frac{9900}{4\pi} = r^2$$

$$\sqrt{\frac{9900}{4\pi}} = r$$

$$28.07 \approx r$$

The radius is approximately 28.1 cm.

**BUILD YOUR SKILLS**

18. Find the surface area of a sphere with a radius of 1.3 m.

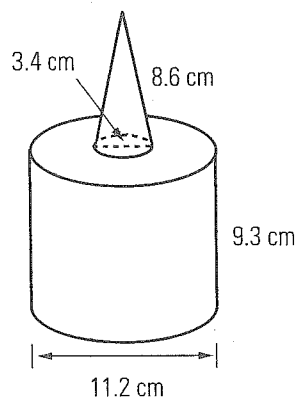
19. Find the surface area of a sphere with a diameter of 24.8 mm.

A hemisphere is  
half of a sphere.

20. Find the surface area of a hemisphere with a radius of 18.5 cm.

**Example 8**

Find the surface area of the composite figure.





**SOLUTION**

The component parts of the surface area of the figure will be the lateral area of the cone plus the surface area of the cylinder minus the circular area of the cone.

Calculate the lateral area of the cone. The diameter is 3.4 cm, so the radius is 1.7 cm.

$$A_1 = \pi r s$$

$$A_1 = \pi(1.7)(8.6)$$

$$A_1 \approx 45.93 \text{ cm}^2$$

Calculate the surface area of the top and bottom of the cylinder. The diameter is 11.2 cm, so the radius is 5.6 cm.

$$A_2 = \pi r^2$$

$$A_2 = \pi(5.6)^2$$

$$A_2 \approx 98.52 \text{ cm}^2$$

Calculate the surface area of the side of the cylinder.

$$A_3 = C \times h$$

$$A_3 = 2\pi r \times h$$

$$A_3 = 2\pi(5.6)(9.3)$$

$$A_3 \approx 327.23 \text{ cm}^2$$

Calculate the area of the base of the cone.

$$A_4 = \pi r^2$$

$$A_4 = \pi(1.7)^2$$

$$A_4 \approx 9.08 \text{ cm}^2$$

Calculate the total surface area of the figure.

SA = surface area of cylinder + surface area of cone – area of base of cone

$$SA = 2(A_2) + A_3 + A_1 - A_4$$

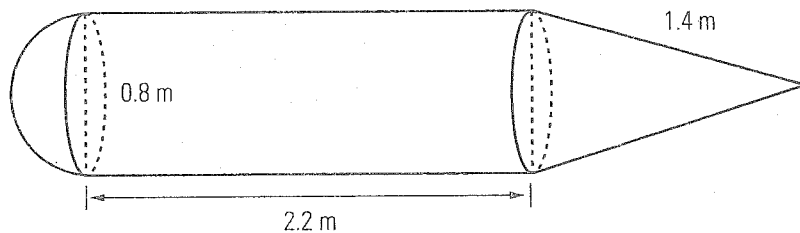
$$SA = 2(98.52) + 327.23 + 45.93 - 9.08$$

$$SA \approx 561.1 \text{ cm}^2$$

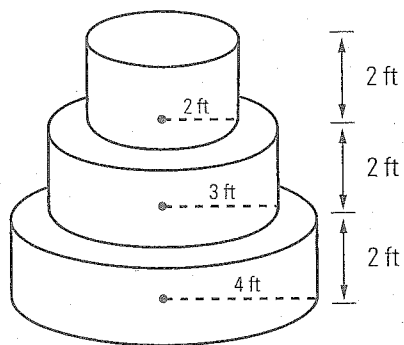
The surface area is approximately 561.1 cm<sup>2</sup>.

**BUILD YOUR SKILLS**

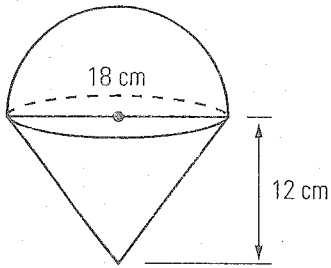
21. Find the surface area the following figure.



22. Three cylinders with radii of 4 feet, 3 feet, and 2 feet are stacked one on top of the other. Each has a height of 2 feet. What is the total exposed surface area?

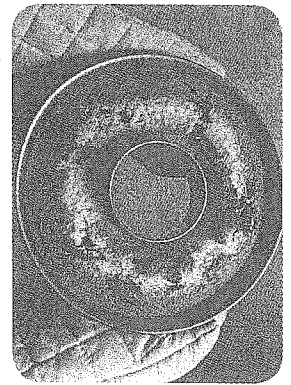
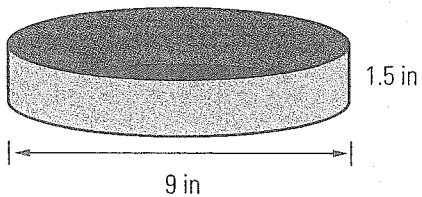


23. Find the surface area of the cone topped by the hemisphere shown in the following diagram.



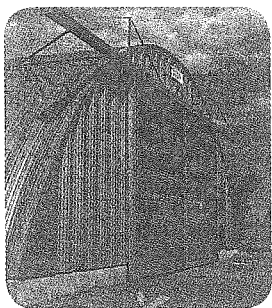
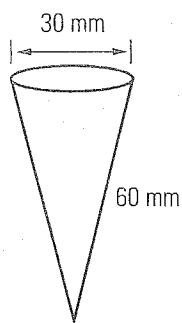
### PRACTISE YOUR NEW SKILLS

1. Bob is a metal worker and is making round cake pans. How much metal will he use in making a 9-inch round cake tin that is  $1\frac{1}{2}$  inches tall?



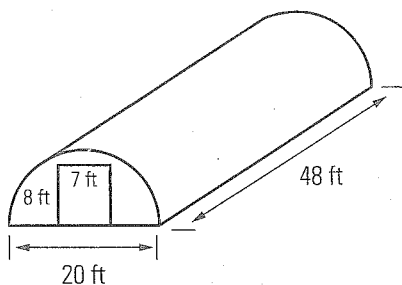
What if the cake pan had a hole in the middle? How would you calculate how much metal was needed?

2. How many square centimetres of paper are needed to produce 50 conical cups like the one below?

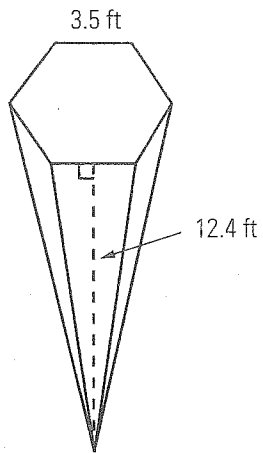


Quonset huts are made of corrugated steel. They are popular because they are easy to assemble and are very durable. They are often used as sheds or garages.

3. How much corrugated steel will be needed to cover a Quonset hut (a kind of building that is half a cylinder) that is 20 feet wide by 48 feet long if both the front and the back are covered, except for a door that is 8 feet tall by 7 feet wide?



4. Find the lateral surface area of the hexagonal pyramid below.



5. Calculate the surface area of a cone that has a radius of 12 feet and a slant height of 15 feet.
6. A tennis ball has a diameter of 6.7 cm. What is its surface area?