

3.4

Volume and Capacity of Spheres, Cones, and Pyramids

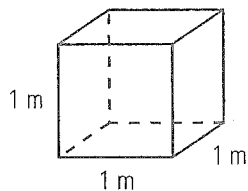
NEW SKILLS: WORKING WITH VOLUME AND CAPACITY OF SPHERES

The volume of a sphere is calculated using the following formula.

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

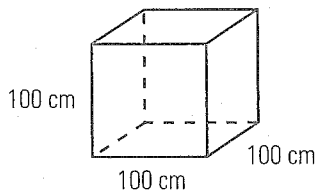
The capacity of a spherical container can be calculated the same as for a prism or cylinder. Start by calculating the volume, then convert to a unit of capacity. Remember that 1000 cm^3 equals 1 L.

You will also need to know how to convert between cubic metres and cubic centimetres. A box with dimensions of 1 m by 1 m by 1 m would have a volume of 1 m^3 .



Convert the box's dimensions to centimetres.

$$1 \text{ m} = 100 \text{ cm}$$



Calculate the volume.

$$V = \ell wh$$

$$V = 100 \times 100 \times 100$$

$$V = 1\,000\,000 \text{ cm}^3$$

Therefore, 1 m^3 is equal to $1\,000\,000 \text{ cm}^3$.

For more details, see page 148 of *MathWorks 11*.

Example 1

A spherical exercise ball has a diameter of 1.2 m.

- What is its volume?
- What is its capacity?

SOLUTION

- Use the formula for the volume of a sphere. The radius is half the diameter, so the radius is 0.6 m.

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \times \pi \times (0.6)^3$$

$$V \approx 0.904\,779 \text{ m}^3$$

The volume of the sphere is about 0.9 m^3 .

- Convert the volume in cubic metres to cubic centimetres.

$$1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$$

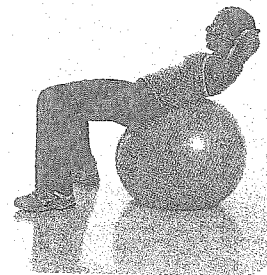
$$0.904\,778\,7 \times 1\,000\,000 = 904\,779 \text{ cm}^3$$

Convert to litres.

$$1000 \text{ cm}^3 = 1 \text{ L}$$

$$904\,779 \div 1000 \approx 905 \text{ L}$$

The capacity is approximately 905 L.



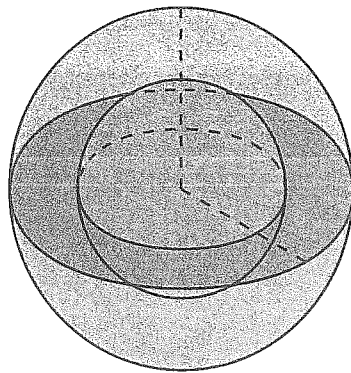
An exercise ball can be a great way to work out your abdominal muscles.

BUILD YOUR SKILLS

- Find the volume of each sphere.
 - A sphere with a radius of 8.5 cm.

b) A sphere with a diameter of 78 cm.

2. A sphere with a radius of 46 cm is centred inside a sphere with a radius of 76 cm.



a) What is the volume of the space between the two spheres?

b) What is the capacity?

NEW SKILLS: WORKING WITH VOLUME AND CAPACITY OF PYRAMIDS

The volume of a pyramid is directly related to the volume of a prism with the same base and height. The volume is calculated using the following formula.

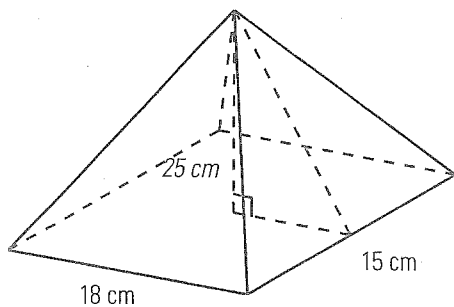
$$V = \frac{1}{3} \times A_{\text{base}} \times h$$

For a rectangular pyramid this can be written as:

$$V = \frac{1}{3} \ell wh$$

Example 2

Calculate the volume and capacity of this pyramid.

**SOLUTION**

Use the formula for calculating the volume of a pyramid.

$$V = \frac{1}{3} \ell wh$$

$$V = \frac{1}{3} \times 15 \times 25 \times 18$$

$$V = 2250 \text{ cm}^3$$

The volume of the pyramid is 2250 cm^3 .

Convert the volume to units of capacity.

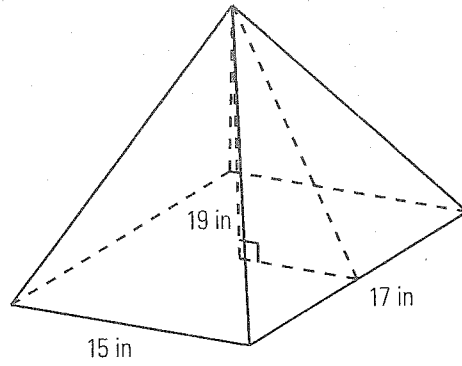
$$2250 \div 1000 = 2.25 \text{ L}$$

The capacity of the pyramid is 2.25 L.

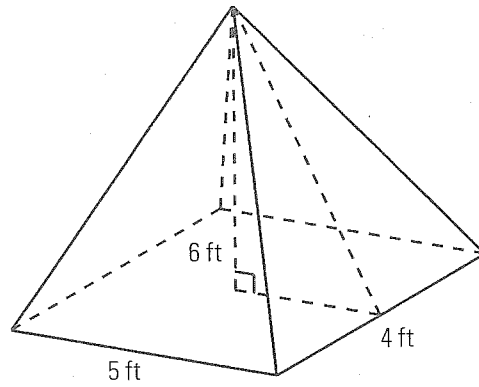
BUILD YOUR SKILLS

3. Find the volume of each pyramid.

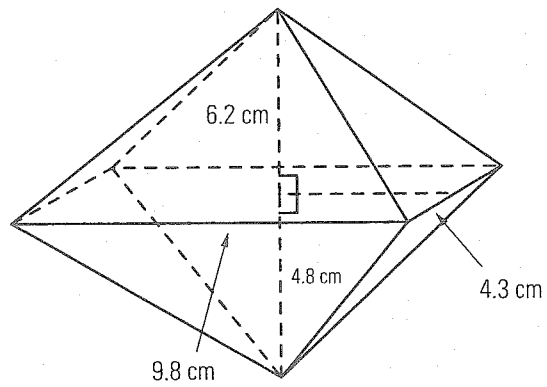
a)



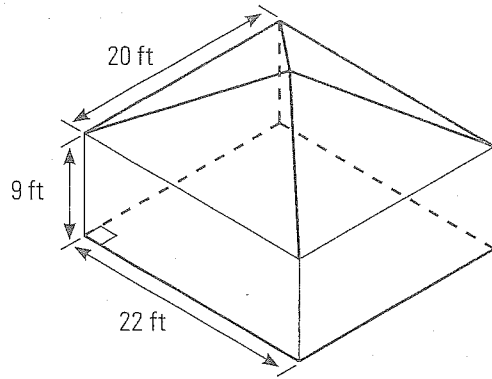
b)



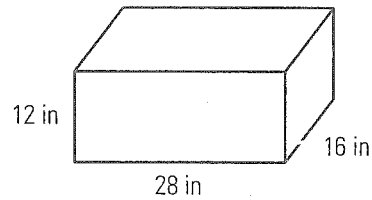
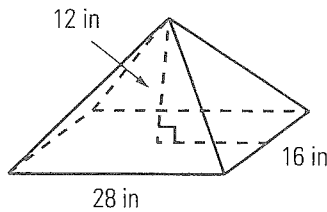
c)



4. What are the volume and capacity of the following figure? The height to the peak is 15 ft.

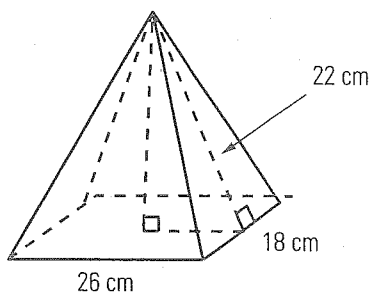


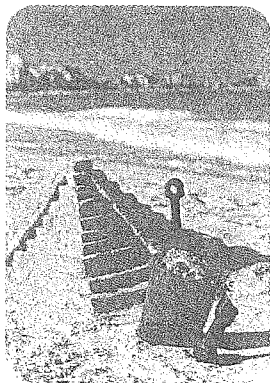
5. Calculate the volume of this prism and pyramid. What is the difference in volume?



Example 3

Calculate the volume of this pyramid.





If you knew the side lengths and height of this sand pyramid, could you estimate how much sand was used to make it?

SOLUTION

You are given the slant height of the pyramid, but not the actual height. Calculate the height using the Pythagorean theorem.

The base of the triangle will be half the width of the pyramid.

$$26 \text{ cm} \div 2 = 13 \text{ cm}$$

$$a^2 + b^2 = c^2$$

$$13^2 + b^2 = 22^2$$

$$b^2 = 22^2 - 13^2$$

$$b = \sqrt{22^2 - 13^2}$$

$$b = \sqrt{484 - 169}$$

$$b = \sqrt{315}$$

$$b \approx 17.7 \text{ cm}$$

Use this height to calculate the volume.

$$V = \frac{1}{3} \ell wh$$

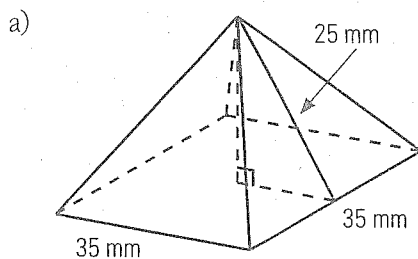
$$V = \frac{1}{3} \times 26 \times 18 \times 17.7$$

$$V \approx 2761.2 \text{ cm}^3$$

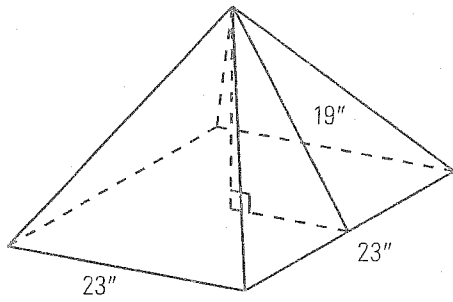
The volume of the pyramid is about 2761.2 cm³.

BUILD YOUR SKILLS

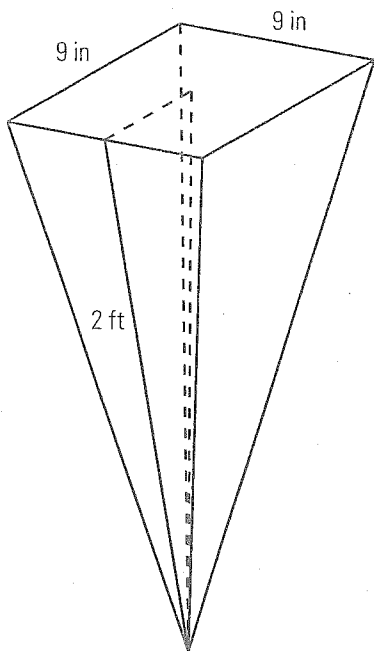
6. Calculate the volume of each of these pyramids.



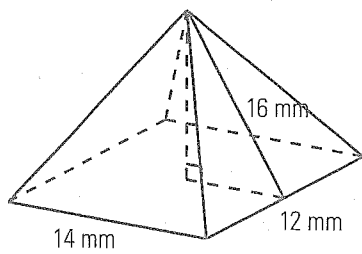
b)



c)



d)



NEW SKILLS: WORKING WITH THE VOLUME AND CAPACITY OF CONES

The volume of a cone is equal to $\frac{1}{3}$ of the volume of a cylinder with the same base and height. As with pyramids, the volume is calculated using the following formula.

$$V = \frac{1}{3} \times A_{\text{base}} \times h$$

Since the base is a circle, the formula can be rewritten as:

$$V = \frac{1}{3} \pi r^2 h$$

Example 4

A paper cup in the shape of a cone has a radius of 3.2 cm and a height of 6 cm. How much water can the cup hold?

SOLUTION

Calculate the volume of the cone.

$$V = \frac{1}{3} \times A_{\text{base}} \times \text{height}$$

$$V = \frac{1}{3} \times \pi r^2 \times h$$

$$V = \frac{1}{3} \pi (3.2)^2 (6)$$

$$V \approx 64.3 \text{ cm}^3$$

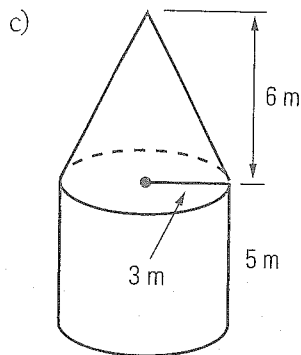
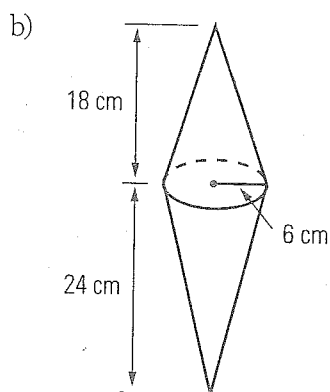
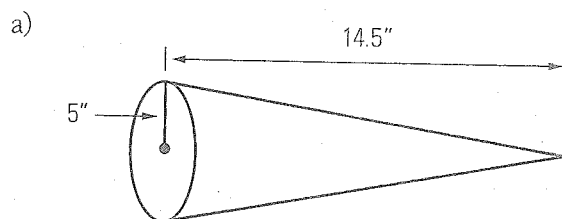
Convert the volume to units of capacity.

$$1000 \text{ cm}^3 = 1 \text{ L or } 1000 \text{ mL}$$

64.3 cm³ equals 64.3 mL. The cup can hold 64.3 mL of water.

BUILD YOUR SKILLS

7. Find the volume of each of the following figures.

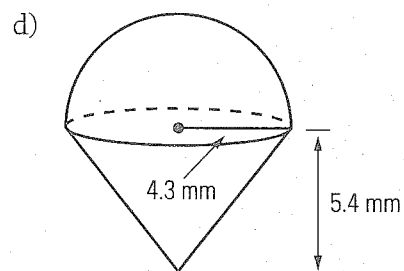
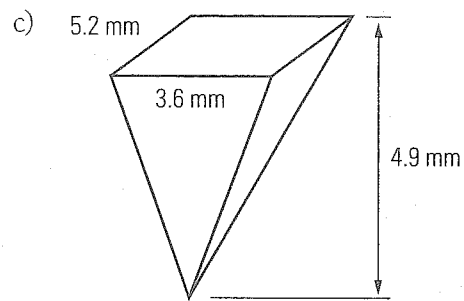
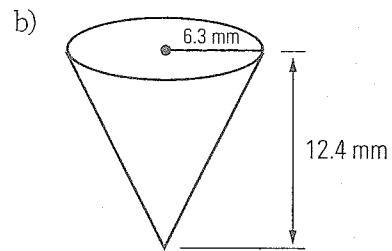
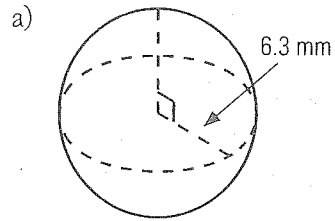


8. A cone has a radius of 12 mm and a volume of 4071.5 mm^3 . What is its height?

9. A cone has a slant height of 15 cm and a radius of 8 cm. Determine its volume.

PRACTISE YOUR NEW SKILLS

1. Find the volume of each of the following figures.

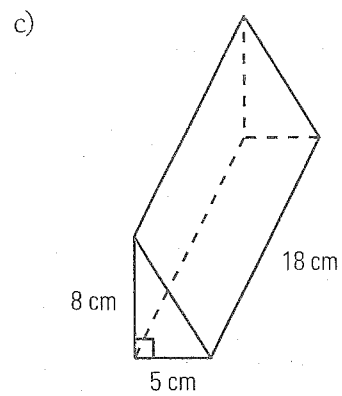
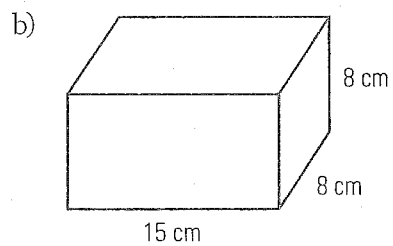
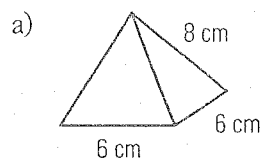


2. What is the capacity in US gallons of a spherical water tower with a diameter of 31.2 feet? 1 cubic foot equals 7.48 US gallons.

3. A square-based pyramid with a base with sides of 14 cm, and its height is 45 cm. A cone has a diameter of 14 cm and also has a height of 45 cm.
- What is the difference in volume between the two figures?
 - What is the difference in capacity?
4. When grain is poured from a hopper onto the ground, it forms a cone-shaped pile. The diameter of the base is approximately 4.5 m and the height is 1.2 m. How many cubic metres of grain are in the pile?
5. Three metal spheres are dropped into a jug of water and sink to the bottom. If the spheres are 3.4 cm, 2.8 cm, and 4.6 cm in radius, what volume of water do they displace?

CHAPTER TEST

1. Draw the net of each figure and calculate its surface area.



2. Calculate surface area and the volume of a box that is 3 m by 2 m by 7.5 m.

3. A cylindrical tin can has a radius of 4.5 cm and a height of 5 cm.

a) What is the surface area of the can?

b) What is its volume?

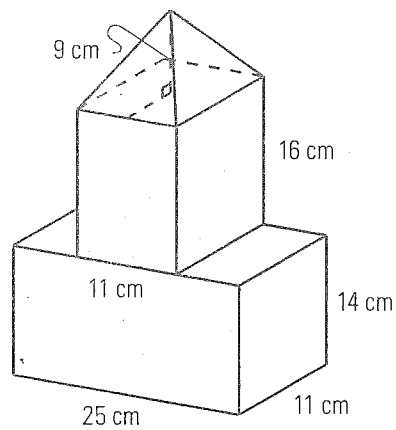
c) What is the can's capacity?

4. A ball has a radius of 56 mm.

a) Calculate the surface area.

b) What is the volume of the sphere?

5. Calculate the total surface area and volume of the following figure.



6. A pile of gravel is conical in shape. If the diameter is approximately 6.8 m and the height is 2.8 m, what is the volume of gravel in the pile?
7. The roof of a building is shaped like a square pyramid, with a base of 18 m by 18 m. If the slant height of the roof is 12 m and roofing costs $\$5.75/\text{m}^2$, how much will it cost for roofing material?