

3.3

Volume and Capacity of Prisms and Cylinders

NEW SKILLS: VOLUME AND CAPACITY OF RECTANGULAR PRISMS

volume: the measure of the space a three-dimensional object occupies

capacity: the amount a three-dimensional object can hold

The **volume** of an object is a measure of the amount of space it occupies. The **capacity** of an object is a measure of how much it can hold. Hollow objects (like a cardboard box) have volume and capacity, while solid objects (like a cement brick) have only volume.

Volume is measured in cubed units, such as m^3 . Capacity is measured in units such as litres or gallons.

Volume and capacity are closely related. In the metric system a volume of 1000 cm^3 is equivalent to a capacity of 1 L.

The volume of both a prism and a cylinder are found by multiplying the area of the base by the height of the object.

$$V = A_{\text{base}} \times h$$

The formula is the same even if the prism/cylinder is oblique. Just remember that the height is always the perpendicular distance between the two bases.

For more details, see page 138 of *MathWorks 11*.

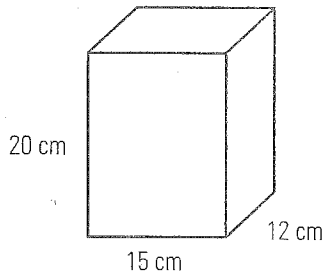
Example 1

A rectangular prism that has a base that is 15 cm by 12 cm and a height of 20 cm.

- Draw a diagram and calculate the volume of the prism.
- Calculate the capacity of the first prism.

SOLUTION

a)



Use the formula for volume.

$$V = A_{\text{base}} \times h$$

$$V = \ell wh$$

$$V = 15 \times 12 \times 20$$

$$V = 3600 \text{ cm}^3$$

b) Since 1 L equals 1000 cm^3 , divide the volume by 1000.

$$3600 \div 1000 = 3.6 \text{ L}$$

The prisms each have a capacity of 3.6 L.

BUILD YOUR SKILLS

1. Find the volume and capacity of the following rectangular prisms.

a) The base is 15.7 cm by 18.8 cm and the height is 12.5 cm.

b) The base is a square with sides of 2.75 m, and the height is 4.5 m.

c) The base is $1\frac{1}{2}$ inches by $3\frac{3}{4}$ inches, and the height is $2\frac{1}{4}$ inches.

Example 2

A rectangular prism has a square base with side lengths of 7 cm. Its volume is 392 cm^3 . Calculate the height of the prism.

SOLUTION

Use the formula for volume and solve for h .

$$V = \ell wh$$

$$392 = 7 \times 7 \times h$$

$$392 = 49h$$

$$\frac{392}{49} = h$$

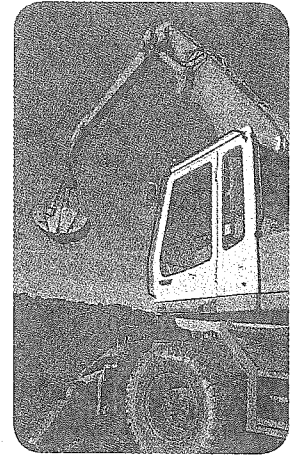
$$8 = h$$

The height of the rectangular prism is 8 cm.

BUILD YOUR SKILLS

- A rectangular prism has a base of 5.2 m by 7.8 m. Its volume is 142 m^3 . What is the height of the prism?
- One rectangular prism has dimensions of 18 cm by 12 cm by 32 cm. A second prism has a base that is 14 cm by 20 cm. Approximately what must its height be if it has the same volume as the first prism?

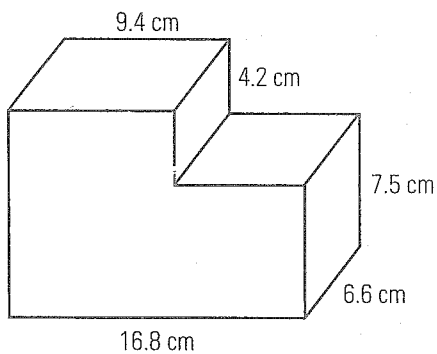
4. Patrice is in charge of the excavation for the foundation of a building. If a hole must be dug that is 35 m by 25 m by 12 m, how many trips will be required to remove the dirt if a trailer can carry only 15 cubic metres of dirt?



Excavators are used to remove dirt and rocks when constructing foundations.

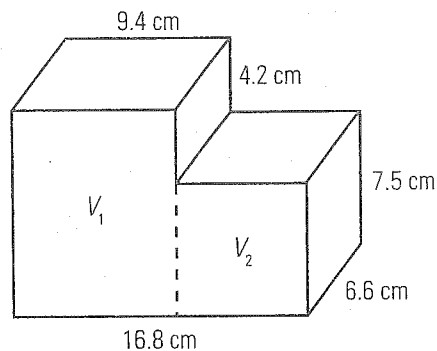
Example 3

Calculate the volume and capacity of the composite prism.



SOLUTION

The volume of this composite prism can be found by dividing it into two rectangular prisms.



$$V_1 = \ell wh$$

$$V_1 = 9.4 \times 6.6 \times (4.2 + 7.5)$$

$$V_1 = 9.4 \times 6.6 \times 11.7$$

$$V_1 \approx 725.9 \text{ cm}^3$$

$$V_2 = \ell wh$$

$$V_2 = (16.8 - 9.4) \times 6.6 \times 7.5$$

$$V_2 = 7.4 \times 6.6 \times 7.5$$

$$V_2 \approx 366.3 \text{ cm}^3$$

Calculate the total volume.

$$V_{\text{total}} = V_1 + V_2$$

$$V_{\text{total}} = 725.9 + 366.3$$

$$V_{\text{total}} = 1092.2 \text{ cm}^3$$

The total volume is 1092.2 cm^3 .

Divide by 1000 to calculate capacity.

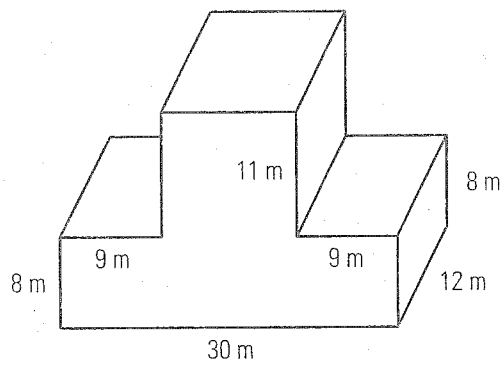
$$1092.2 \div 1000 = 1.0922 \text{ L}$$

Its capacity is about 1.1 L.

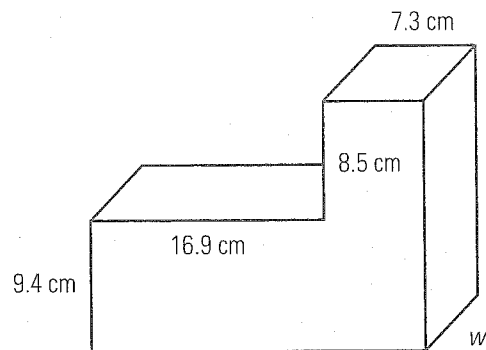
BUILD YOUR SKILLS

5. a) The volume of the figure in Example 3 could have been calculated by dividing the composite prism into two different prisms than the ones used in the solution given. Draw a diagram to indicate this. Label it with its dimensions.
- b) The volume could also have been calculated by subtraction. Draw a diagram to indicate how you could do this. Label the diagram with its dimensions.

6. Find the volume of this figure.



7. The capacity of this composite prism is 1.8 litres. Determine its depth, w .



NEW SKILLS: WORKING WITH THE VOLUME OF CYLINDERS

The volume of a prism is calculated using the following formula.

$$V = A_{\text{base}} \times h$$

The area of a the base of a cylinder can be calculated as:

$$A = \pi r^2$$

If you combine these two formulas, the formula for the volume of a cylinder is as follows.

$$V = \pi r^2 h$$

Example 4

A can of tomato sauce has a radius of 3.8 cm and a height of 10.2 cm.

- What is the volume of the can?
- How much tomato sauce (in litres) does the can hold?

SOLUTION

- Start by calculating the volume of the can.

$$V = \pi r^2 h$$

$$V = \pi(3.8)^2(10.2)$$

$$V \approx 462.7 \text{ cm}^3$$

- 1000 cm³ equals 1 L, so divide the volume by 1000.

$$462.7 \div 1000 = 0.4627 \text{ L}$$

The can holds about 0.5 L of tomato sauce.

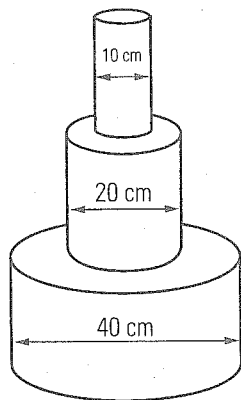


Delicious home-made sauce recipes are sometimes kept as secret family recipes, passed down through generations.

BUILD YOUR SKILLS

8. Calculate the volume and capacity of a cylinder with a diameter of 15 cm and a height of 36 cm.

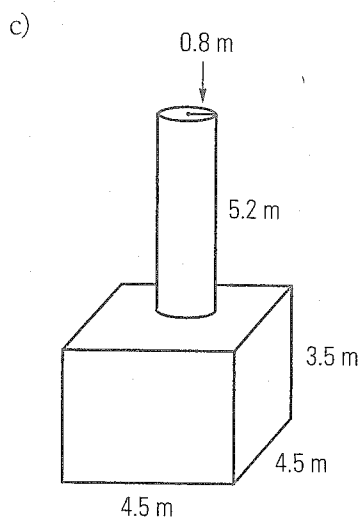
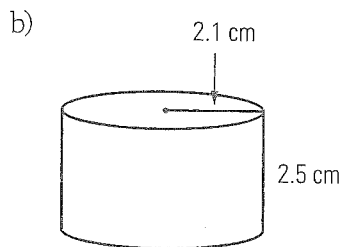
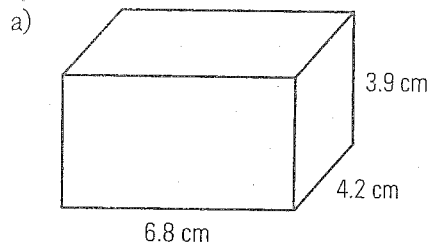
9. Calculate the volume and capacity of the stacked cylinders below. Each cylinder has a height of 20 cm.



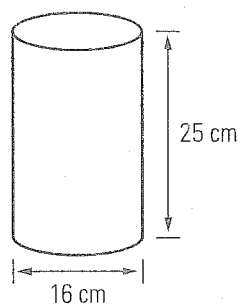
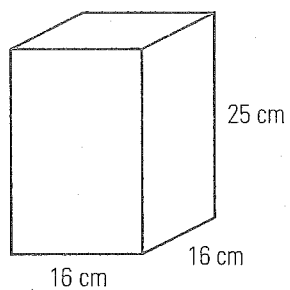
10. A large tin can has a capacity of 3.24 L. If the can has a diameter of 15.56 cm. What is the height of the can?

PRACTISE YOUR NEW SKILLS

1. Calculate the volume of each figure.



2. Which of these figures has the larger capacity? Show your work.



3. A silo has a diameter of 24 feet and is filled to a height of 70 feet. What is the volume of grain stored in it?

4. Graydon is laying a cement patio. It is to be 6 m long by 4.5 m wide, and 15 cm deep.
- Calculate the volume of concrete needed to fill the patio area.
 - If concrete weighs approximately 2400 kg per cubic metre, how much will the patio concrete weigh?
5. A fish tank in the shape of a rectangular prism contains 15 L of water. If the base of the tank is 30 cm by 20 cm, what is the depth of the water?
6. A large cylindrical fuel storage tank has a capacity of 20 000 L. If it has a diameter of 2.4 m, what is the height of the tank?