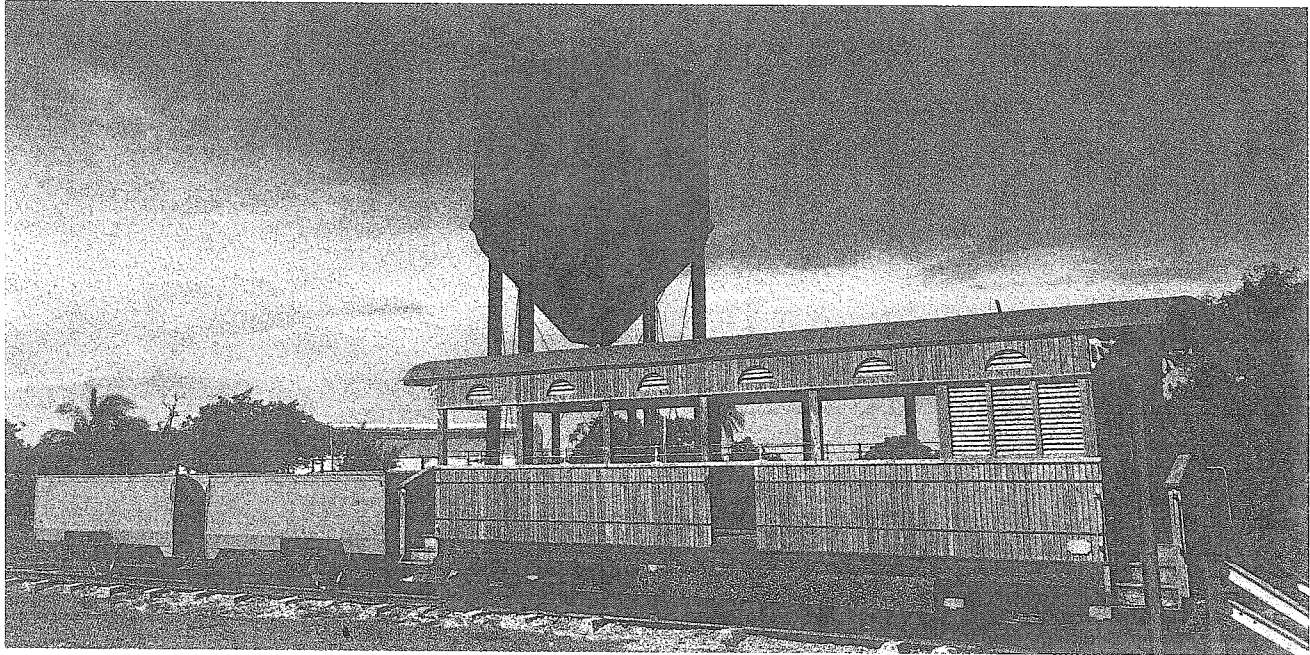


# Chapter 3

---

## Surface Area, Volume, and Capacity



*How much water do you think this water tank can hold? What would you need to know to calculate the exact amount?*

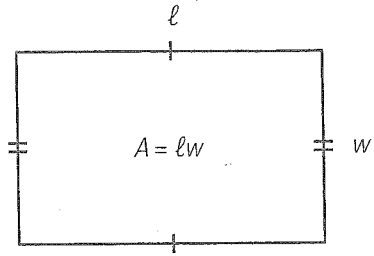
### 3.1

## Surface Area of Prisms

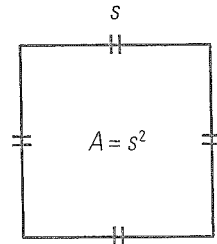
---

### REVIEW: WORKING WITH AREAS OF TWO-DIMENSIONAL FIGURES

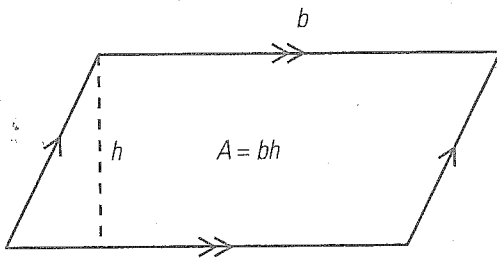
In this chapter, you will need to know how to calculate the area of various two-dimensional figures, including squares, rectangles, parallelograms, triangles, and circles. The following diagrams show how to calculate the areas of each of these shapes.



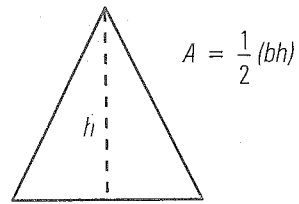
Rectangle



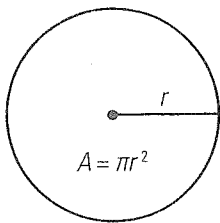
Square



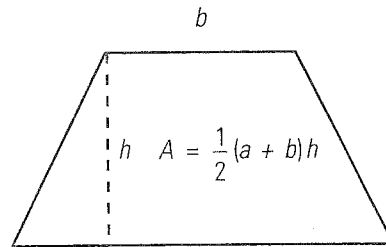
Parallelogram



Triangle



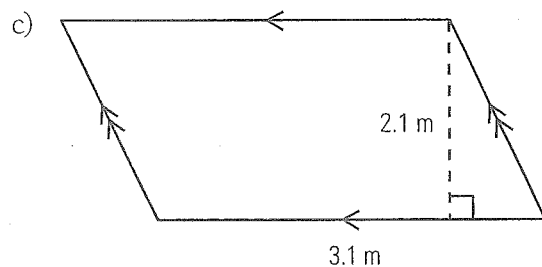
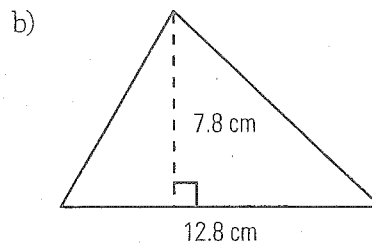
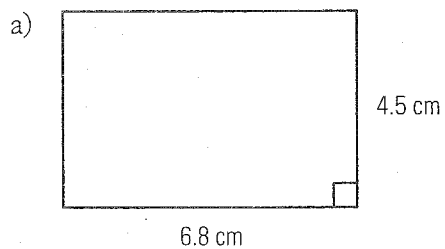
Circle



Trapezoid

**Example 1**

Calculate the area of each figure.

**SOLUTION**

- a) Use the formula for calculating the area of a rectangle.

$$A = \ell w$$

$$A = 6.8 \times 4.5$$

$$A = 30.6 \text{ cm}^2$$

- b) Calculate the area of the triangle.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 12.8 \times 7.8$$

$$A = 49.92 \text{ cm}^2$$

- c) Calculate the area of the parallelogram.

$$A = bh$$

$$A = 3.1 \times 2.1$$

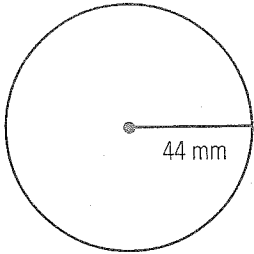
$$A = 6.51 \text{ m}^2$$

Remember that, for any figure, the height is always perpendicular (at right angles) to the base.

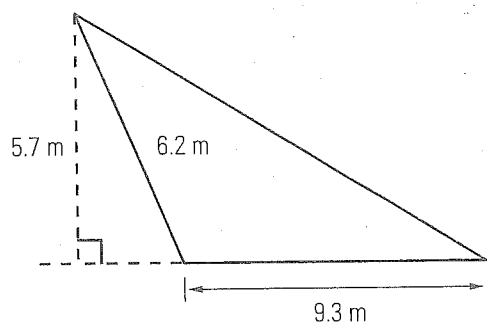
**BUILD YOUR SKILLS**

1. For each picture, name the shape and calculate the area.

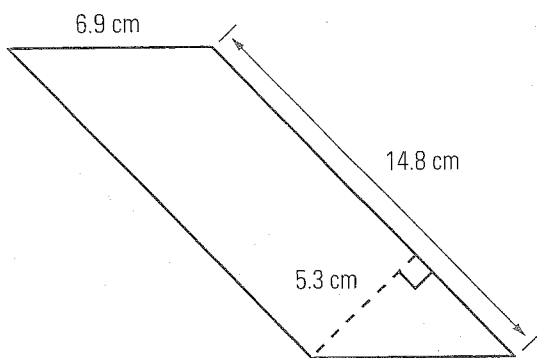
a)



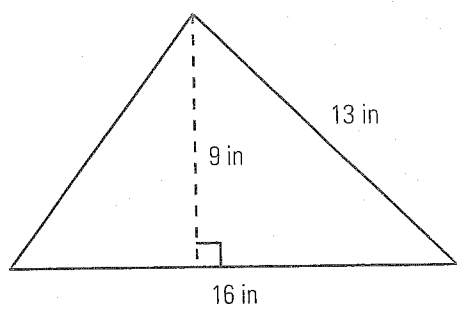
b)



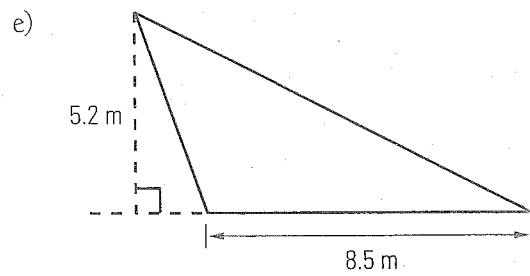
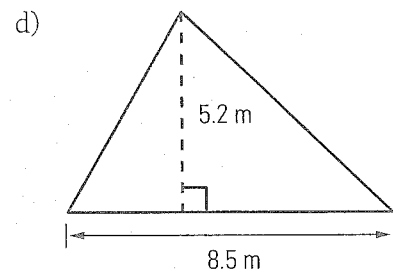
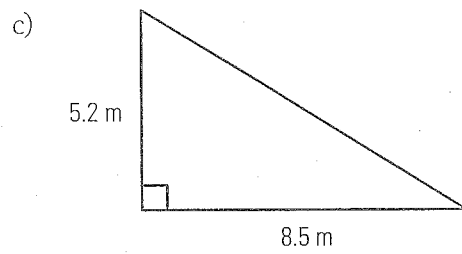
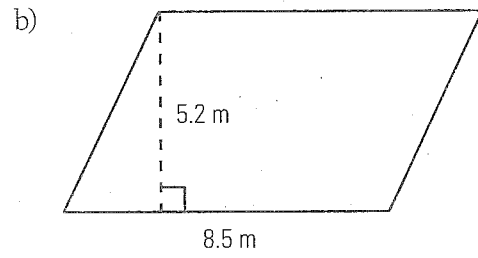
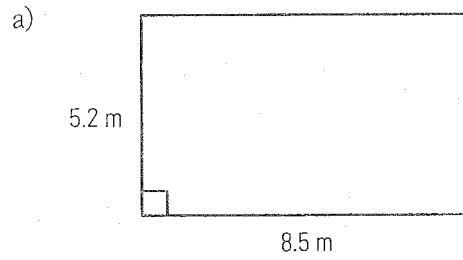
c)



d)



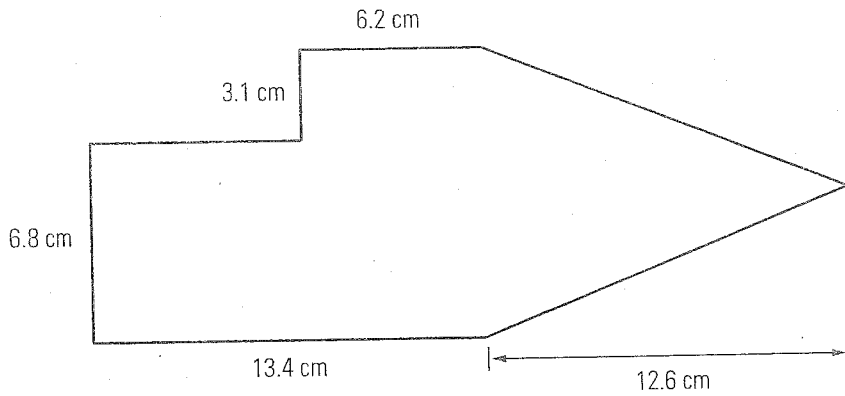
2. For each picture, name the shape and calculate the area.



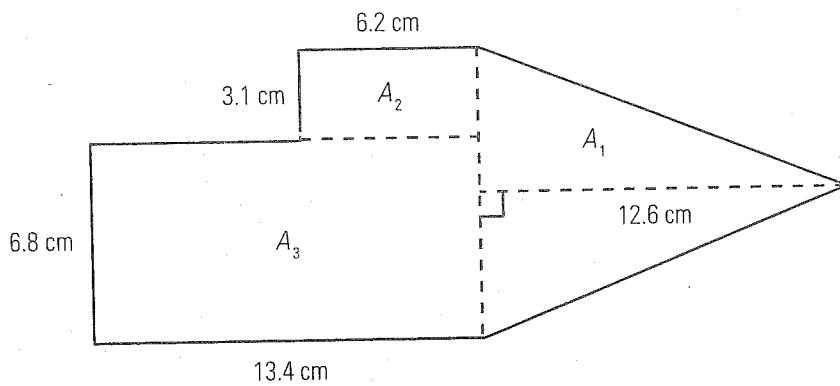
f) What do you notice about the area calculations?

**Example 2**

Find the area of the following figure.

**SOLUTION**

To calculate the area, you will need to divide the figure into regular shapes. This figure can be divided into a triangle and two rectangles. The dimensions will be as indicated. Find the area of each part and add them together.



Start by calculating the area of the triangle.

$$A_1 = \frac{1}{2}bh$$

$$A_1 = \frac{1}{2} \times 9.9 \times 12.6$$

$$A_1 = 62.37 \text{ cm}^2$$

Label the area calculations with subscript numbers ( $A_1$ ,  $A_2$ ,  $A_3$ ) to help you keep track of the shapes.

Next, calculate the area of the smaller rectangle.

$$A_2 = \ell w$$

$$A_2 = 6.2 \times 3.1$$

$$A_2 = 19.22 \text{ cm}^2$$

Calculate the area of the second rectangle.

$$A_3 = \ell w$$

$$A_3 = 13.4 \times 6.8$$

$$A_3 = 91.12 \text{ cm}^2$$

Add the areas of the shapes to calculate the total area.

$$A_{\text{total}} = A_1 + A_2 + A_3$$

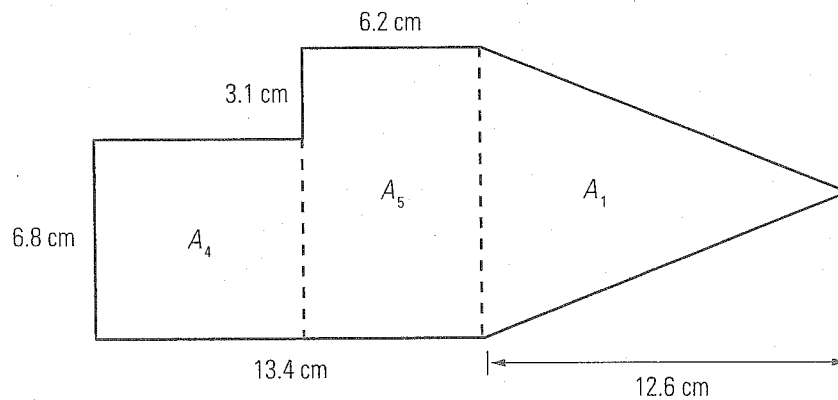
$$A_{\text{total}} = 62.37 + 19.22 + 91.12$$

$$A_{\text{total}} = 172.71 \text{ cm}^2$$

The total area is  $172.71 \text{ cm}^2$ .

#### ALTERNATIVE SOLUTION

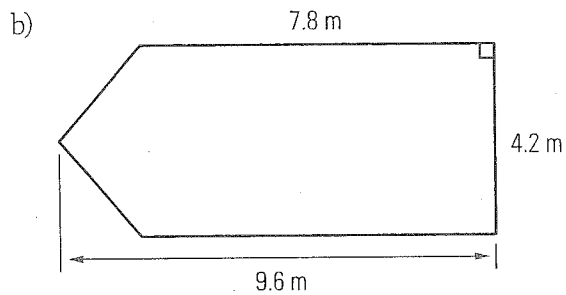
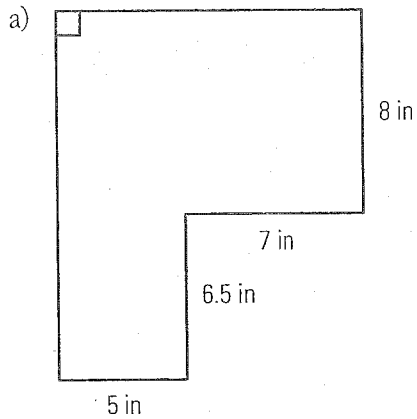
The figure could have been divided into different regular shapes.



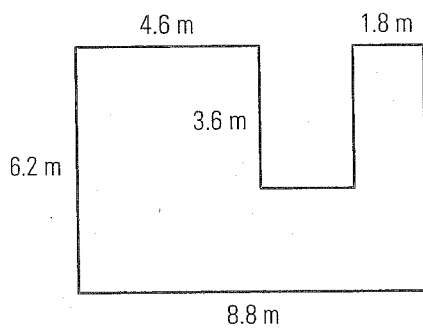
You would calculate the area of the triangle,  $A_1$ , as above. You would then find the areas of the two rectangles,  $A_4$  and  $A_5$ .

**BUILD YOUR SKILLS**

3. Find the areas of the following figures.



4. Show four different ways you could divide the figure below to calculate its area. Show all measurements. Choose one method to calculate the area.



A carpet installer needs to know how to calculate the surface area of irregularly shaped rooms, in order to know how much carpet will be needed.



## NEW SKILLS: WORKING WITH THE SURFACE AREA OF PRISMS

**prism:** a 3-D shape with ends that are congruent, parallel polygons and sides that are parallelograms

**base:** one of the parallel faces of a prism

**lateral face:** a face that connects the bases of a prism

A **prism** is a three-dimensional object with:

- ends, called **bases**, that are congruent and parallel, and
- sides, called **lateral faces**, that are parallelograms.

The prism is a right prism if the sides are perpendicular to the bases. The lateral faces will be rectangles.

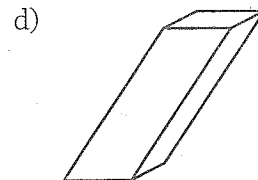
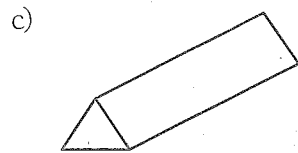
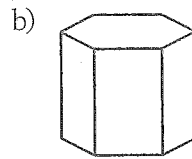
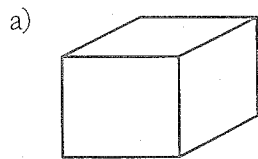
If the lateral faces are not perpendicular to the base, it is an oblique prism and the sides will be parallelograms.

A prism is named by the shape of its base and whether it is right or oblique.

For more details, see page 118 of *MathWorks 11*.

### Example 3

Name the following prisms.



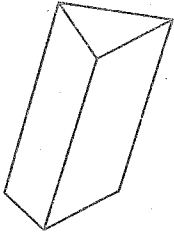
### SOLUTION

- a) This is a right rectangular prism. The base is a rectangle and the lateral faces are perpendicular to it.
- b) This is a right hexagonal prism. The base is a hexagon and the lateral faces are perpendicular to it.
- c) This is a right triangular prism. The base is a triangle and the lateral faces are perpendicular to it.
- d) This is an oblique rectangular prism. The base is a rectangle and the lateral faces are not perpendicular to it.

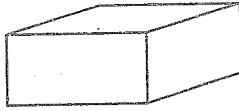
**BUILD YOUR SKILLS**

5. Name the following prisms.

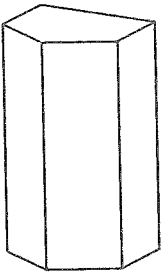
a)



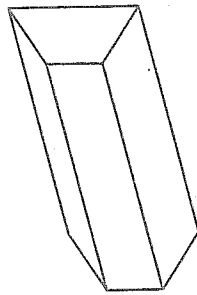
b)



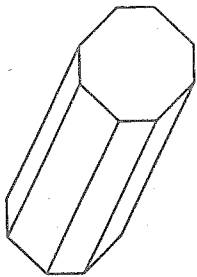
c)



d)



e)



f)



<i>Prism</i>	<i>Shape of base</i>	<i>Right or oblique</i>	<i>Shape of lateral faces</i>	<i>Name of prism</i>
a)				
b)				
c)				
d)				
e)				
f)				

## NEW SKILLS: WORKING WITH NETS

**net:** 2-D pattern that can be folded to make a 3-D shape

**surface area:** the area covered by the outside surfaces of a three-dimensional shape

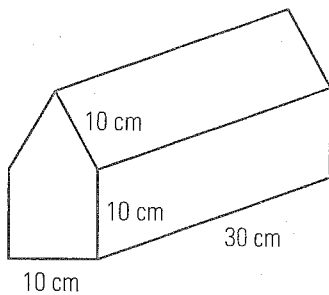
A **net** is a two-dimensional pattern that can be folded to form a three-dimensional shape. Think of a pizza box: it is made up of one piece of cardboard, folded into the shape of a right rectangular prism.

The **surface area** of a prism is the area that it would take up if it were laid out flat, as in its net.

For more details, see page 117 of *MathWorks 11*.

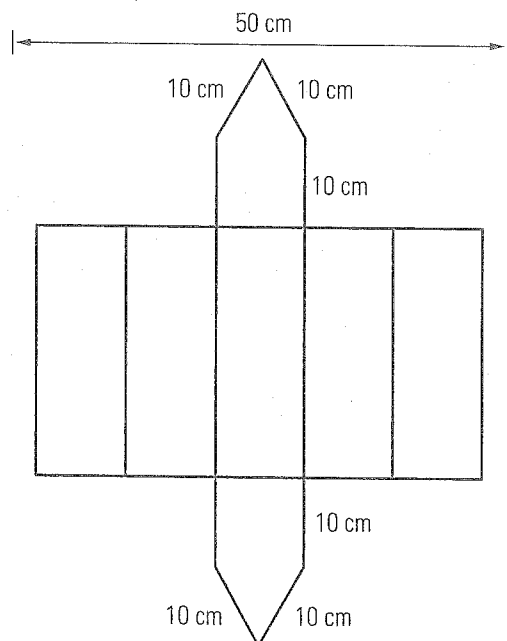
### Example 4

If this right pentagonal prism were made from one piece of cardboard, what would the piece of cardboard look like?



### SOLUTION

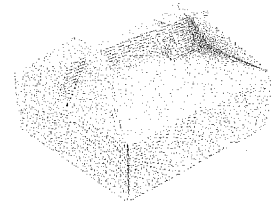
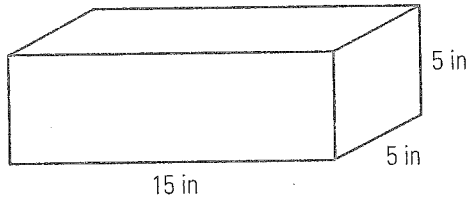
The net of the prism would look like the following diagram.



**BUILD YOUR SKILLS**

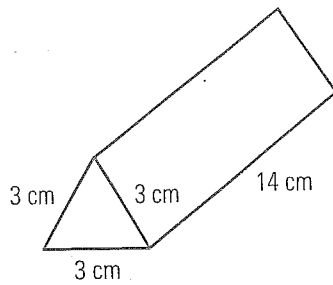
6. Draw nets for the following prisms, and label the dimensions of each side.

a)

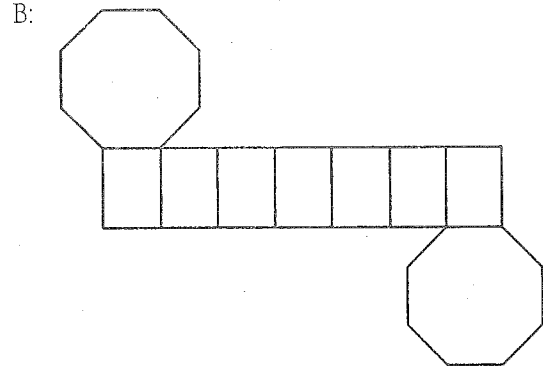
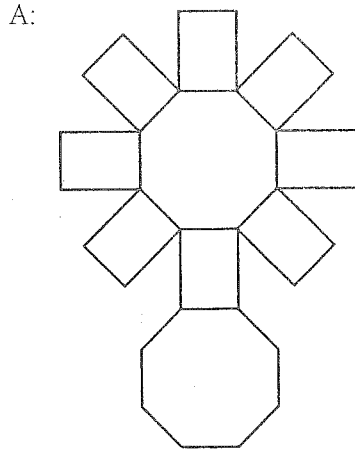


Can you imagine what the net of this box might look like?

b)

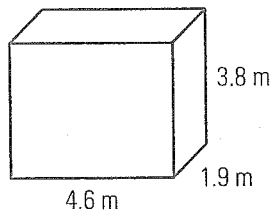


7. Ralph says that diagram A is the net of a right octagonal prism. Manon disagrees. She says that diagram B is the correct net for a right octagonal prism. Who is correct? Justify your answer.



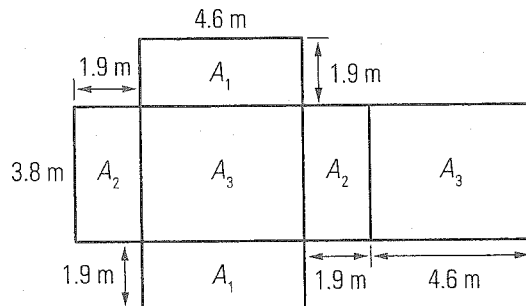
**Example 5**

Find the surface area of the right rectangular prism given below.



**SOLUTION**

Draw a net of the prism.



Calculate the area of each of the parts of the net.

There are two rectangles that are 1.9 m by 4.6 m (labelled as  $A_1$ ).

$$A_1 = \ell w$$

$$A_1 = 1.9 \times 4.6$$

$$A_1 = 8.74 \text{ m}^2$$

There are two rectangles that are 1.9 m by 3.8 m (labelled as  $A_2$ ).

$$A_2 = \ell w$$

$$A_2 = 1.9 \times 3.8$$

$$A_2 = 7.22 \text{ m}^2$$

There are two rectangles that are 4.6 m by 3.8 m (labelled as  $A_3$ ).

$$A_3 = \ell w$$

$$A_3 = 4.6 \times 3.8$$

$$A_3 = 17.48 \text{ m}^2$$

Calculate the total surface area.

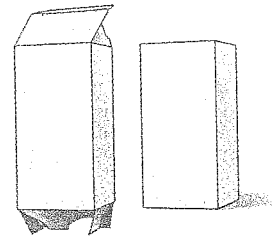
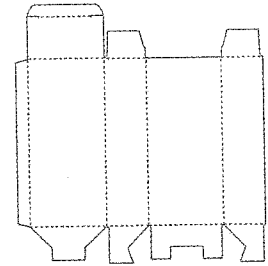
$$SA = 2(A_1) + 2(A_2) + 2(A_3)$$

$$SA = 2(8.74) + 2(7.22) + 2(17.48)$$

$$SA = 17.48 + 14.44 + 34.96$$

$$SA = 66.9 \text{ m}^2$$

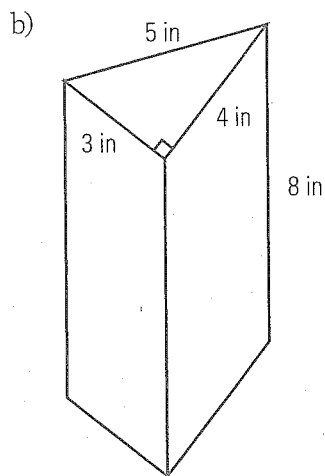
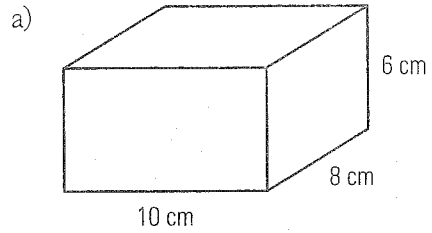
The surface area of the prism is  $66.9 \text{ m}^2$ .

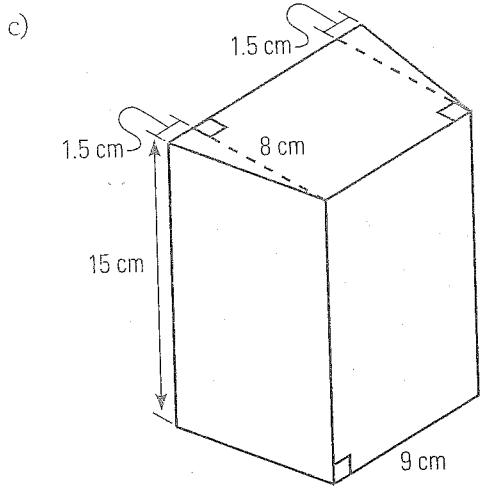


Nets are sometimes used in assembly instructions, as shown in this illustration for a box manufacturer.

**BUILD YOUR SKILLS**

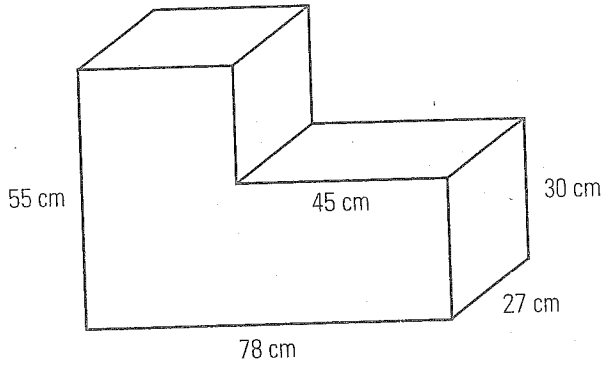
8. For each diagram, draw a net and use it to calculate surface area.





**Example 6**

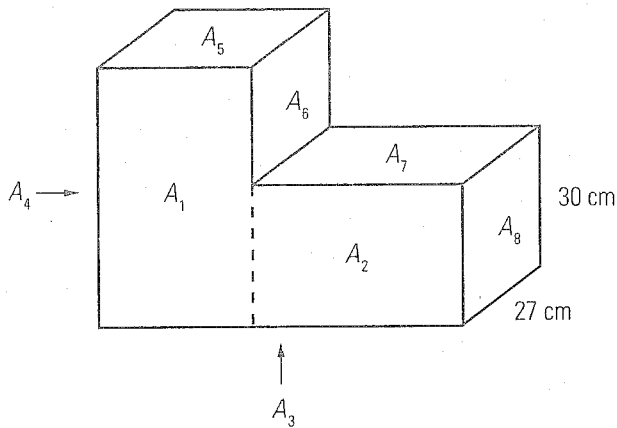
Find the surface area of this figure.





**SOLUTION**

Divide the figure into parts.



$$A_1 = \ell w$$

$$A_1 = (78 - 45) \times 55$$

$$A_1 = 1815 \text{ cm}^2$$

$$A_5 = \ell w$$

$$A_5 = (78 - 45) \times 27$$

$$A_5 = 891 \text{ cm}^2$$

$$A_2 = \ell w$$

$$A_2 = 45 \times 30$$

$$A_2 = 1350 \text{ cm}^2$$

$$A_6 = \ell w$$

$$A_6 = (55 - 30) \times 27$$

$$A_6 = 675 \text{ cm}^2$$

$$A_3 = \ell w$$

$$A_3 = 78 \times 27$$

$$A_3 = 2106 \text{ cm}^2$$

$$A_7 = \ell w$$

$$A_7 = 45 \times 27$$

$$A_7 = 1215 \text{ cm}^2$$

$$A_4 = \ell w$$

$$A_4 = 55 \times 27$$

$$A_4 = 1485 \text{ cm}^2$$

$$A_8 = \ell w$$

$$A_8 = 30 \times 27$$

$$A_8 = 810 \text{ cm}^2$$

Calculate the total surface area.

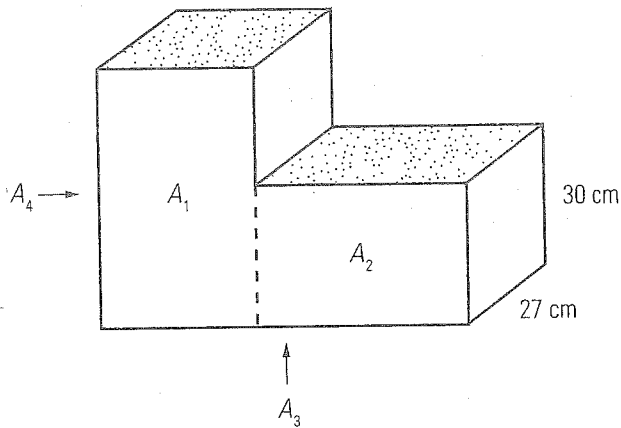
$$SA = 2(A_1) + 2(A_2) + A_3 + A_4 + A_5 + A_6 + A_7 + A_8$$

$$SA = 2(1815) + 2(1350) + 2106 + 1485 + 891 + 675 + 1215 + 810$$

$$SA = 13\,512 \text{ cm}^2$$

**ALTERNATIVE SOLUTION**

If you look carefully at the diagram, you will discover that you don't need to calculate the area of each individual surface.



The shaded portions are equal in area to the left side of the figure (the end that you can't see,  $A_4$ ). The polka-dotted parts are equal in area to the bottom of the figure,  $A_3$ . You will need to divide the front and back into two rectangles as indicated.

Calculate the areas of the different parts as in the solution above, but you only need to calculate parts  $A_1$ ,  $A_2$ ,  $A_3$ , and  $A_4$ .

$$A_1 = 1815 \text{ cm}^2$$

$$A_2 = 1350 \text{ cm}^2$$

$$A_3 = 2106 \text{ cm}^2$$

$$A_4 = 1485 \text{ cm}^2$$

Calculate the total surface area.

$$SA = 2(A_1) + 2(A_2) + 2(A_3) + 2(A_4)$$

$$SA = 2(1815) + 2(1350) + 2(2106) + 2(1485)$$

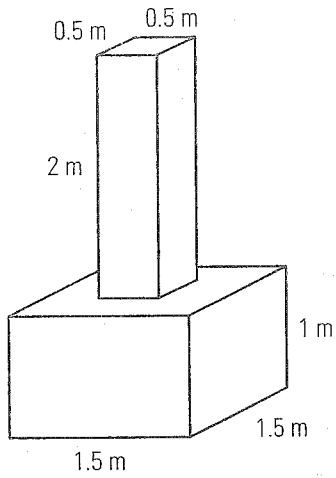
$$SA = 3630 + 2700 + 4212 + 2970$$

$$SA = 13\,512 \text{ cm}^2$$

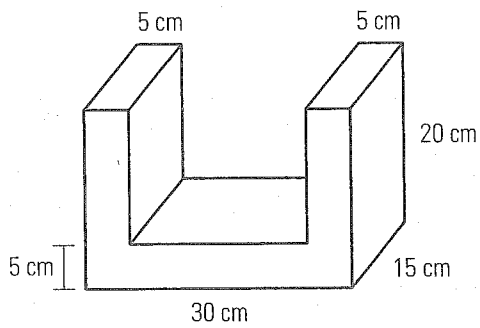
The total surface area is  $13\,512 \text{ cm}^2$ .

**BUILD YOUR SKILLS**

9. Calculate the surface area of the figure below.



10. Find the surface area of the figure below.



**Example 7**

Kareem has been hired to paint the walls and ceiling of a living room in a house. The room is 22.5 feet long, 13.5 feet wide, and 8.5 feet high. There is one window that is 10.5 feet by 6 feet, two windows that are 3.5 feet by 2.5 feet, and two doors that are 2.5 feet by 8 feet.

- What surface area must he paint?
- One gallon of paint covers approximately 255 sq. ft. How many gallons will he have to buy?
- If paint costs \$55.40 per gallon and he wants to make a profit of about \$225.00, how much should he charge to paint the room?

**SOLUTION**

- Find the total surface area of the walls and ceiling and subtract the areas of the windows and doors.

The ceiling is 22.5 feet by 13.5 feet.

$$A_1 = \ell w$$

$$A_1 = 22.5 \times 13.5$$

$$A_1 = 303.75 \text{ sq. ft.}$$

There are two walls that are 22.5 feet by 8.5 feet.

$$A_2 = \ell w$$

$$A_2 = 22.5 \times 8.5$$

$$A_2 = 191.25 \text{ sq. ft.}$$

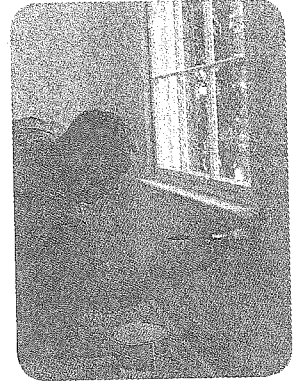
There are two walls that are 13.5 feet by 8.5 feet.

$$A_3 = \ell w$$

$$A_3 = 13.5 \times 8.5$$

$$A_3 = 114.75 \text{ sq. ft.}$$

Calculate the total surface area of the walls.



It is important to accurately calculate the surface area to be painted, so that you can make sure you buy enough paint to complete the project.

$$SA_1 = A_1 + 2(A_2) + 2(A_3)$$

$$SA_1 = 303.75 + 2(191.25) + 2(114.75)$$

$$SA_1 = 303.75 + 382.5 + 229.5$$

$$SA_1 = 915.75 \text{ sq. ft.}$$

There is one window that is 10.5 feet by 6 feet.

$$A_4 = \ell w$$

$$A_4 = 10.5 \times 6$$

$$A_4 = 63 \text{ sq. ft.}$$

There are two windows that are 3.5 feet by 2.5 feet.

$$A_5 = \ell w$$

$$A_5 = 3.5 \times 2.5$$

$$A_5 = 8.75 \text{ sq. ft.}$$

There are two doors that are 2.5 feet by 8 feet.

$$A_6 = \ell w$$

$$A_6 = 2.5 \times 8$$

$$A_6 = 20 \text{ sq. ft.}$$

Calculate the surface area that does not need painting.

$$SA_2 = A_4 + 2(A_5) + 2(A_6)$$

$$SA_2 = 63 + 2(8.75) + 2(20)$$

$$SA_2 = 63 + 17.5 + 40$$

$$SA_2 = 120.5 \text{ sq. ft.}$$

Calculate the area to be painted.

$$SA_{\text{paint}} = SA_1 - SA_2$$

$$SA_{\text{paint}} = 915.75 - 120.5$$

$$SA_{\text{paint}} = 795.25 \text{ sq. ft.}$$

The area to be painted is 795.25 square feet.

- b) Each can of paint covers approximately 255 square feet, and Kareem has 795.25 square feet to cover. Divide the total surface area by the area per can.

$$795.25 \div 255 \approx 3.1 \text{ gallons}$$

Kareem will need to buy 4 gallons of paint.

- c) Multiply the number of gallons by the cost per gallon.

$$4 \times \$55.40 = \$221.60$$

If Kareem wants to make a profit of about \$225.00, add this amount to the cost of the paint.

$$\$221.60 + \$225.00 = \$446.60$$

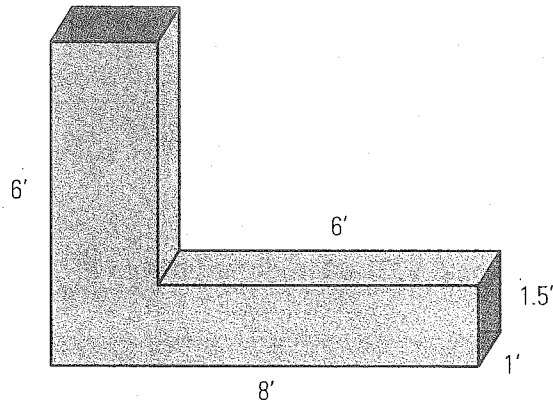
Kareem will need to charge about \$446.60 to paint the room. He might round this up to \$450.00.

### BUILD YOUR SKILLS

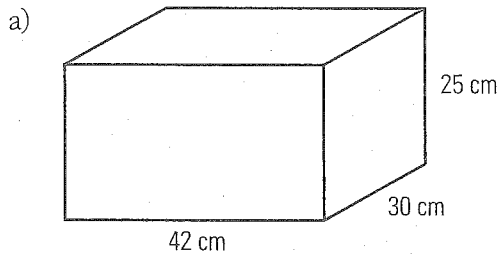
11. Aamir is building a storage chest in the shape of a rectangular prism. The chest will be 90 cm long, 70 cm deep, and 60 cm high.
- a) What will be the outer surface area of the box?

- b) Aamir needs to buy 20% more wood than the surface area, to account for wastage during cutting. What area of wood will he need to buy?

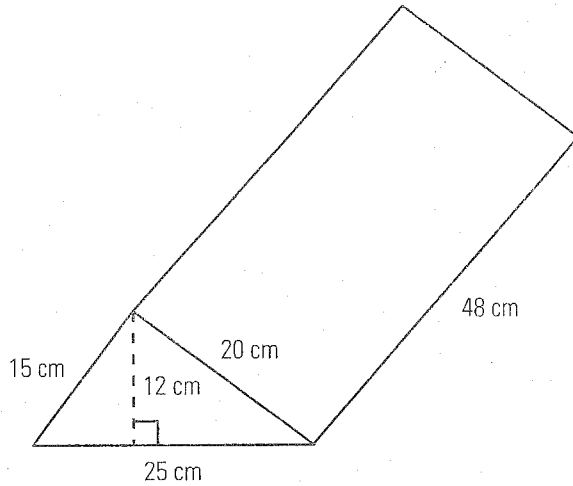
12. Shar is installing an L-shaped heating duct in a house. If the duct has the measurements shown in the diagram below, what will be the total surface area of the duct? The ends of the duct are open.

**PRACTISE YOUR NEW SKILLS**

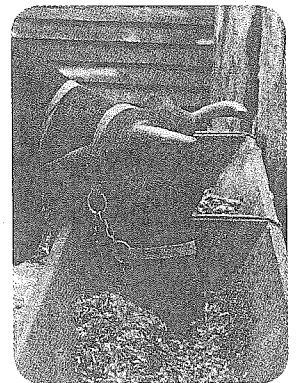
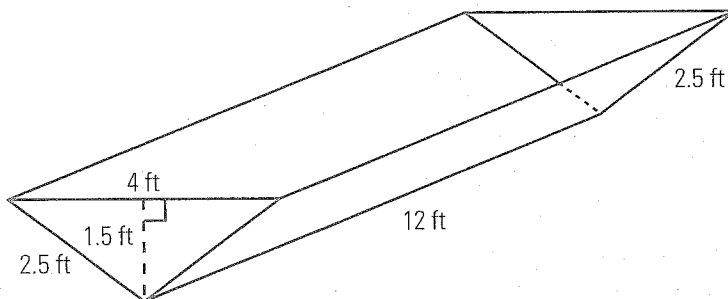
1. Sketch the nets of the prisms and use them to determine their surface areas.



b)



2. Hoeth is building a feed trough for his cattle. The trough is to be triangular with a length of 12 feet, a height 1.5 feet, and a width of 4 feet. How much wood is required to build it?



Modern cattle troughs are often made out of durable plastics or metal.

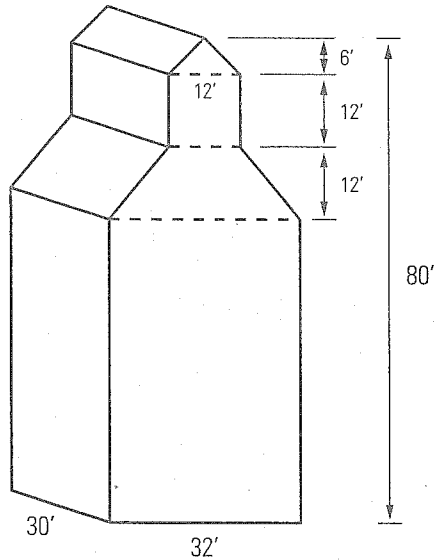


3. Aaron is going to construct a fish tank that is 1.2 m long, 0.6 m wide, and 0.4m high. How much glass will he need to make it? (Note: There will be no glass on the top.)



4. A grain elevator is a tower containing a bucket elevator, which scoops up grain and deposits it into a storage facility.

Calculate the surface area of the grain elevator shown here. (Assume that the front face of the grain elevator is vertically symmetrical.)



Grain elevators were traditionally build of wood, but they are being replaced by more modern ones made of steel or concrete. Some of the old wooden structures are being preserved as heritage sites.